SIMPLE METHOD OF MEASURING A VARIABLE HEAT FLUX

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A method of measuring a time-dependent heat flux is described and results of a verification are presented.

A number of methods [1] can be used to measure a heat flux which varies with time.

Methods were described in [2-7] for determining large variable heat fluxes from a plasma jet to an obstacle. The heat-flux sensors were copper rods modeling a semiinfinite body or a flat plate. Thermocouples were mounted along the rod at one or several cross sections, and one thermocouple was placed close to the heated surface. In particular, in using the semiinfinite body method [5], the distances of the thermocouple from the working surface of the sensor should optimally be about 1 mm. From the measured temperatures one can recover the heat flux to the sensor surface, using some appropriate correlation.

The measurement of temperature at several sections along a heated surface (in particular, close to the surface) complicates sensor design and data reduction and reduces sensor reliability.

The sensor design can be simplified by locating the thermocouple at the rear face of the rod. This structure is found in the exponential (or calorimetric) heat-flux sensor [8]. The exponential method can also be used to measure unsteady heat flux at small values of Bi [1]. It is difficult to use it for large heat fluxes because of the condition mentioned. In addition, in the derivation of the theoretical relation

$$q = c\rho R\, dt/d\tau \tag{1}$$

the dependence of the thermophysical properties of the calorimeter material on temperature, which may be substantial, has not been accounted for. For copper, in particular, the variation in properties from their mean values can reach 20-30%. Thus, the use of Eq. (1), even to determine a constant heat flux, requires further verification.

Similarly, the dependence of material property on temperature is not accounted for in a number of methods for measuring unsteady heat fluxes.

The problem of heating a flat plate with an arbitrary heat flux on one surface, with the second surface thermally insulated, and allowing for the dependence of properties on temperature, can be formulated as follows:

$$c(t)\rho\frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(t)\frac{\partial t}{\partial x}\right), \tag{2}$$

$$c = c_0 + c_1 t; \quad \lambda = \lambda_0 + \lambda_1 t, \tag{3}$$

$$q|_{x=R} = q(\tau), \tag{4}$$

$$q|_{x=0} = 0, \tag{5}$$

$$t|_{\tau=0} = 0. \tag{6}$$

It is known that the temperature field over a plate with constant heat flux, omitting a small initial range, is described by the parabolic relation [9]

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$$t(x, \tau) = \frac{q_c}{\lambda}\left(\frac{a\tau}{R^2} - \frac{R^2 - 3x^2}{6R}\right). \tag{7}$$

We seek a solution of Eqs. (2)-(6) in the form

$$t(x, \tau) = M + Nx^2, \tag{8}$$

where M and N are functions of time. From condition (4) we find

$$q(\tau) = -2RN(\lambda_0 + \lambda_1(M + NR^2)). \tag{9}$$

Expression (8) is automatically satisfied by condition (5). Substituting Eq. (8) into Eq. (2), for $x = 0$, we obtain

$$N = \frac{(c_0 + c_1 M)\rho M'}{2(\lambda_0 + \lambda_1 M)}. \tag{10}$$

Here M and M' are values of the temperature and its time derivative on the rear surface ($x = 0$). By measuring the temperature on the rear surface we can calculate the heat flux from Eqs. (9) and (10). The error associated with approximating the temperature field with the parabola of Eq. (8) can be reduced substantially by applying the calorimetric approach. From Eq. (2) we obtain

$$q(R, \tau) = \int_0^R c\rho \frac{\partial t}{\partial \tau}\, dx = \rho\left[(c_0 + c_1 M)\left(M' + \frac{1}{3} R^2 N'\right) R + R^3 N c_1 \left(\frac{1}{3} M' + \frac{1}{5} N' R^2\right)\right]. \tag{11}$$

For $c_1 = \lambda_1 = 0$ and $q = \text{const}$, $N' = 0$, since $\partial t/\partial \tau$ is independent of x in this case for $\text{Fo} \geqslant 0.3$ [9]. Equation (11) can be transformed to the form

$$q = \rho c R M'; \tag{12}$$

i.e., we obtain the well-known relation (1) of the calorimetric method.

Substituting Eq. (8) into Eq. (2), for $x = R$ we obtain

$$N' = \frac{2N(\lambda_0 + M\lambda_1 + 3N\lambda_1 R^2)}{R^2 \rho\, [c_0 + c_1(M + NR)]} - \frac{M'}{R^2}. \tag{13}$$

Thus, from the measured temperature at the rear surface of the plate we can use Eqs. (10), (13), and (11) to calculate an arbitrarily variable or constant heat flux at the surface $x = R$. The method accounts for the dependence of the thermophysical properties of the sensor material on temperature.

To check the method, a numerical solution using an explicit scheme was performed for the above problem, with several variants for the variation of the heat flux on the surface. The heat flux was evaluated from the temperatures determined using the above formulas.

In the solution we approximated Eq. (2) by a difference equation of the form

$$\theta_{i,k+1} = \theta_{i,k}\left[1 - \frac{2\left(1 + \frac{\lambda_1}{\lambda_0}\theta_{i,k}\right)}{\left(1 + \frac{c_1}{c_0}\theta_{i,k}\right)} \Delta\text{Fo}\right] + \frac{\left(1 + \frac{\lambda_1}{\lambda_0}\theta_{i,k}\right)}{\left(1 + \frac{c_1}{c_0}\theta_{i,k}\right)} \Delta\text{Fo}(\theta_{i+1,k} + \theta_{i-1,k}) +$$

$$+ \frac{\lambda_1/\lambda_0}{\left(1 + \frac{c_1}{c_0}\theta_{i,k}\right)} \cdot \frac{\Delta\text{Fo}}{4}(\theta_{i+1,k} - \theta_{i-1,k})^2; \quad \theta = t - t_0;\ \Delta\text{Fo} = a_0 \Delta\tau/\Delta x^2. \tag{14}$$

The step sizes $\Delta\tau$ and Δx were chosen from the condition

$$0 \leqslant \left[1 - \frac{2\left(1 + \frac{\lambda_1}{\lambda_0}\theta_{i,k}\right)}{1 + \frac{c_1}{c_0}\theta_{i,k}} \Delta\text{Fo}\right] \leqslant 1. \tag{15}$$

It follows from boundary condition (5) that $\theta_{i,k} = \theta_{i-1,k}$. The following variants of the relation $q(\tau)$ were considered, in particular, at the surface $x = R$:

a constant heat flux

$$\theta_{s1,k} = \theta_{2,k} + \frac{q_0 \Delta x}{\lambda_0 + \lambda_1 \theta_{2,k}}, \tag{16}$$

a rectangular variation of heat flux (Fig. 2)

$$\theta_{s1,k} = \theta_{2,k} + \frac{(q_0 + q' k\Delta\tau)\,\Delta x}{\lambda_0 + \lambda_1 \theta_{2,k}}, \tag{17}$$

where the coefficients q_0 and q' are changed at a certain time. The rectangular pulse is a model of the variation of q at the stagnation point of a body during entry into dense atmospheric layers [11] and during heating of a body by plasma jets with fluctuating parameters.

Calculation on the Élektronika-S50 have shown that with a step size of $\Delta x = 0.01$–0.02 cm ($\Delta\tau = 0.25\cdot10^{-4}$–$10^{-4}$ sec) the accuracy is satisfactory (Fig. 1). The program provided for up to 70 points to be calculated along the coordinate.

In determining the heat flux the time derivative of temperature (13) is calculated from the following relations:

for the first point

$$\theta_0' = \frac{1}{2\Delta\tau}(-3\theta_0 + 4\theta_1 - \theta_2) \tag{18}$$

and for the remaining points

$$\theta_k' = (\theta_{k+1} - \theta_{k-1})/2\Delta\tau. \tag{19}$$

The interval $\Delta\tau$ in Eqs. (18) and (19) could be chosen arbitrarily. In contrast with the semiinfinite body method [5], for which each successive heat-flux value is calculated from all the previous values, when one uses Eqs. (10), (11), and (13) the calculation for any time value can be performed independently of the previous heat-flux values. Because of the peculiarity mentioned, a calculation using the method of [5] often leads to steadily increasing oscillations in the results, because of accumulation of errors.

The program for computing heat flux using Eqs. (10), (11), (13), (18), and (19) can easily be run on small type Élektronika-S50 computers.

The results of determining heat flux with an initial constant q are satisfactory (Fig. 2a). The error is 3%. The figure also shows the results of calculating the heat flux by the exponential method, using average values of c and ρ. It should be noted that, in spite of the substantial change in the temperature of the calorimeter element (from room temperature to fusion temperature), the results of calculating using Eq. (1) are in satisfactory agreement (within 10%) with the initial heat-flux values. The temperature curves (Fig. 2a) have linear sections, and the temperature gradients there are practically independent of the

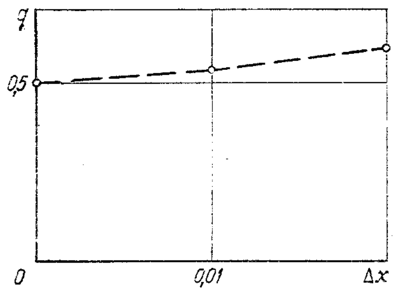

Fig. 1. Resulting heat flux (kW/cm²) as a function of coordinate step size (cm) used in calculating the temperature field; $\tau = 0.1$ sec, initial heat flux $q = q_0 + q'\tau$, where $q' = 5$; kW/cm²/sec, $q_0 = 0$.

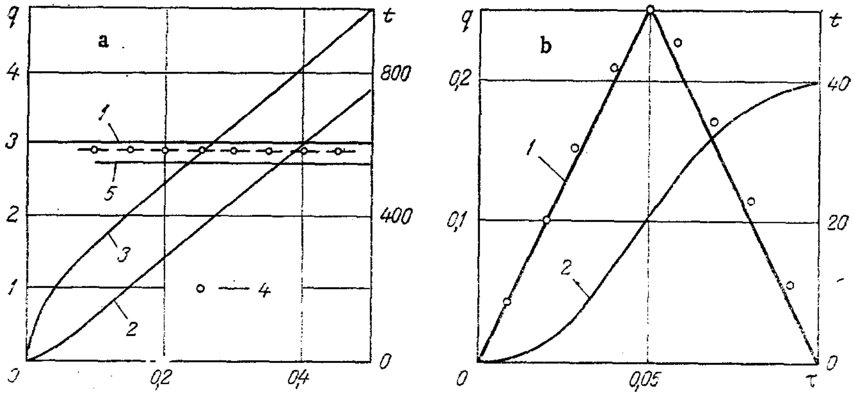

Fig. 2. Heat-flux results (kW/cm²) as a function of time (sec): 1) initial heat flux; 2, 3) temperature at x = R and 0, respectively; 4, 5) resulting heat flux [4) from Eqs. (10), (11) and (13); 5) from Eq. (1)].

coordinate. Thus, the basic conditions of the exponential method are satisfied, and it can be used to measure constant heat-flux values using copper calorimeters, right up to the fusion temperature.

Satisfactory agreement was also obtained with a time-dependent heat flux, both in phase and in amplitude of the calculated results, using the method described, with initial values of q, in spite of the kink in the original dependence (Fig. 2b).

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